Seat No. : _______________

AB-148

April-2018

B.Sc., Sem.-VI

307 : Statistics
(Distribution Theory – II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry equal marks.

(2) Use of scientific calculator is allowed.

(3) Statistical & logarithmic tables and graph papers will be provided on request.

1. (a) State two parameter Cauchy Distribution. For Cauchy distribution, derive mode.

OR

In usual notations, prove that the distribution function of Cauchy distribution is

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x-\mu}{\lambda}\right)$$

(b) Define log normal distribution. Derive moment generating function of log normal distribution.

OR

State the probability density function of Laplace distribution. Derive first two raw moments of Laplace distribution.

2. (a) If two random variables (X, Y) follow bivariate normal distribution, define marginal and conditional distributions. Also, in usual notations, show that Y follows normal distribution.

OR

In usual notations, for bivariate normal distribution, show that

$$E(Y/X) = \mu_2 + \rho\, \sigma_2 (X - \mu_1) / \sigma_1$$

(b) Derive moment generating function of bivariate normal distribution.

OR

If X and Y have bivariate normal distribution with parameters (3, 1, 16, 25, 0.6), determine probabilities : (i) $P[3 < Y < 8]$, (ii) $P[3 < Y < 8 / X = 7]$

3. (a) In usual notations, state and prove Chebyshev's inequality.

OR

If the r.v. having probability mass function as $P(X) = \begin{cases} 2^{-x} & , x = 1, 2, 3, 4 \ldots\ldots \\ 0 & , \text{otherwise} \end{cases}$

Determine the probability $P[\,|x - E(x)| \leq 2]$ and actual probability $P[\,|x - E(x)| \leq 2]$

(b) In usual notations, state and prove Bernoulli's law of large numbers.

OR

Examine whether the law of the large numbers for the sequence $\{x_k\}$ of independent random variables defined as :

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$

$P(X_k = 0) = 1 - 2^{-2k}$

4. (a) State and prove Lindberg-Levy Central Limit Theorem.

OR

If $X_1, X_2, \ldots, X_n$ are poisson variates with parameter m = 2, use the central limit theorem to determine $P[120 < S_n < 160]$, where $S_n = X_1 + X_2 + \ldots\ldots + X_n$ and n = 75.

(b) State uses of central limit theorem and state Liapounff's form of central limit theorem.

OR

The guaranteed average life of a certain type of electric bulbs is 1000 hours with a standard deviation of 125 hours. It is decided to sample the output so as to ensure 90% of the bulbs do not fall short fall of the guaranteed average life by more than 2.5%. Use central limit theorem to find the minimum sample size.

5. Answer the following in brief :

(a) State use of chebyshev's inequality. State general form of chebyshev's inequality.

(b) State different forms of convergence.

(c) State weak law of large numbers. Also, state its use.

(d) State the other name of Laplace distribution. State the characteristic function of Laplace distribution.

(e) Let Z_1 and Z_2 be two independent N(0,1) random variables. Find correlation co-efficient

$\rho(X, Y)$, if $X = Z_1$, and $Y = \rho Z_1 + \sqrt{(1 - \rho^2)}\, Z_2$.

(f) If chebyshev's inequality for a random variable X with standard deviation 3, is $P[6 < X < 18] \geq 3/4$, then find mean of X.

(g) For Cauchy distribution, moments do not exist. Justify your answer.